Growth Model

Endogenous Growth Model

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Overview

1. Solow Model
2. Exogenous vs. Endogenous growth models
3. Endogenous Growth model
4. Conclusions and takeaways

Reference: Williamson, Macroeconomics 5th Edition, Chapter 7-8

Review questions

When there is a technological progress, what happens to the steady state income per capita in Malthusian model VS. Solow model?

Is this consistent with what we see in data, or observe in the real world?

Solow model

Review questions

- Why do we care about the steady state?
- What does the golden rate saving ratio tell you?

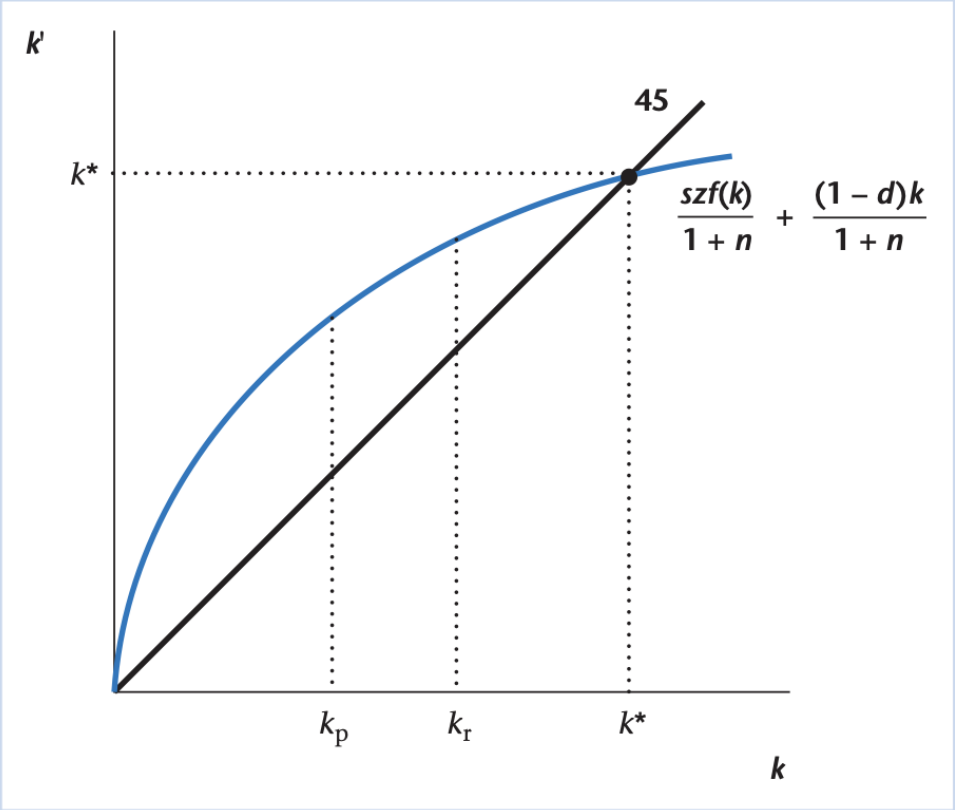
Solow Model

Technology and steady state

What is the Solow model prediction on rich and poor country steady state?

Figure 8.1 Rich and Poor Countries and the Steady State

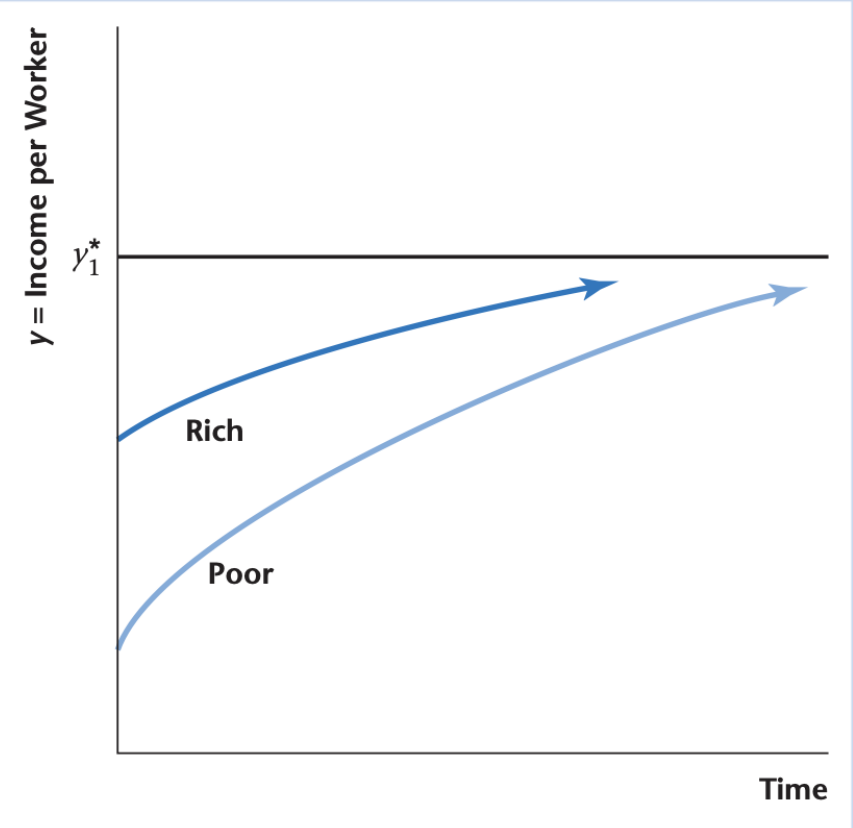
Two otherwise identical countries have initial capital stocks per worker of k_p (the poor country) and k_r (the rich country). Both countries converge in the long-run steady state to the quantity k^* of capital per worker.



Convergence between rich and poor

Figure 8.2 Convergence in Income per Worker Across Countries in the Solow Growth Model

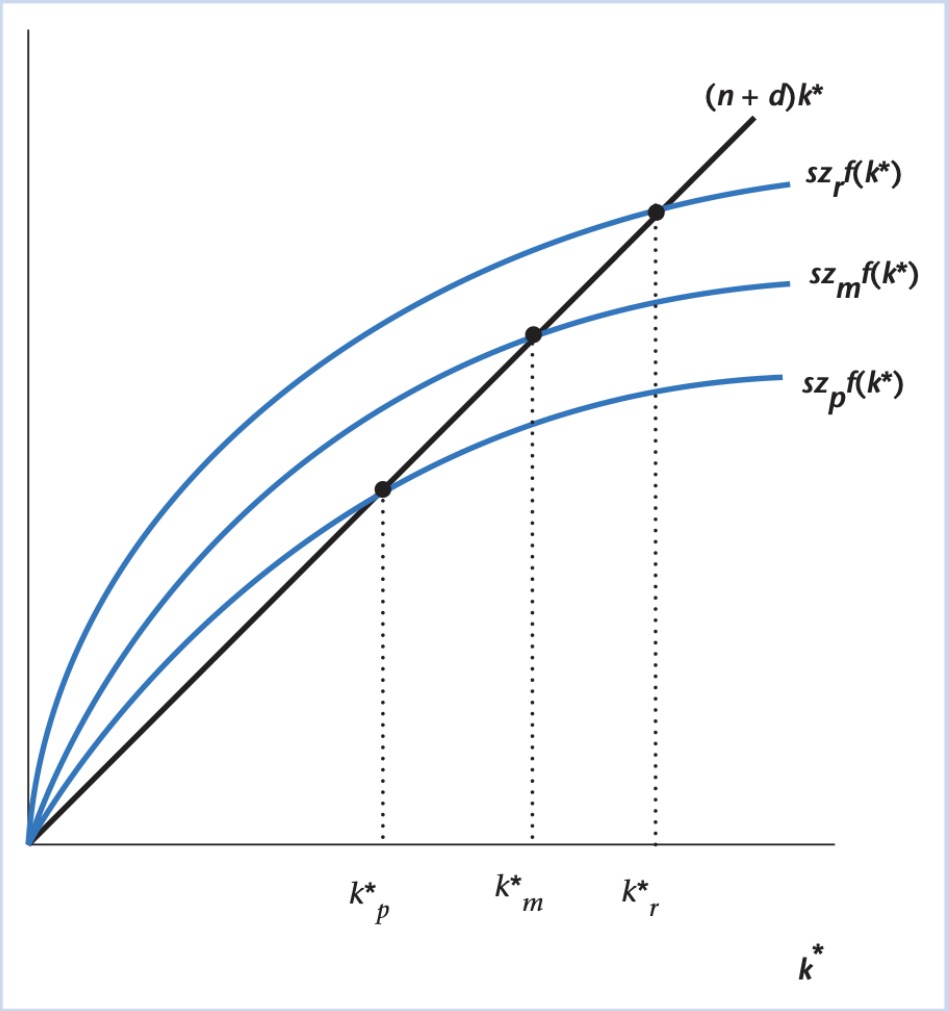
Two otherwise identical countries, one with lower income per worker (the poor country) than the other (the rich country), both converge in the long-run steady state to the same level of income per worker, y_1^*.



Income disparity in the world

Figure 8.4 Differences in Total Factor Productivity Can Explain Disparity in Income per Worker Across Countries

If countries have different levels of total factor productivity due to differing barriers to technology adoption, then capital per worker and income per worker differ across countries in the steady state.



Technological differences

Technology could be one of the reasons explaining income disparity across countries.

But why countries have different level of technology/ or technology adoption? Economics and legal structure could be boundary to technology adoption

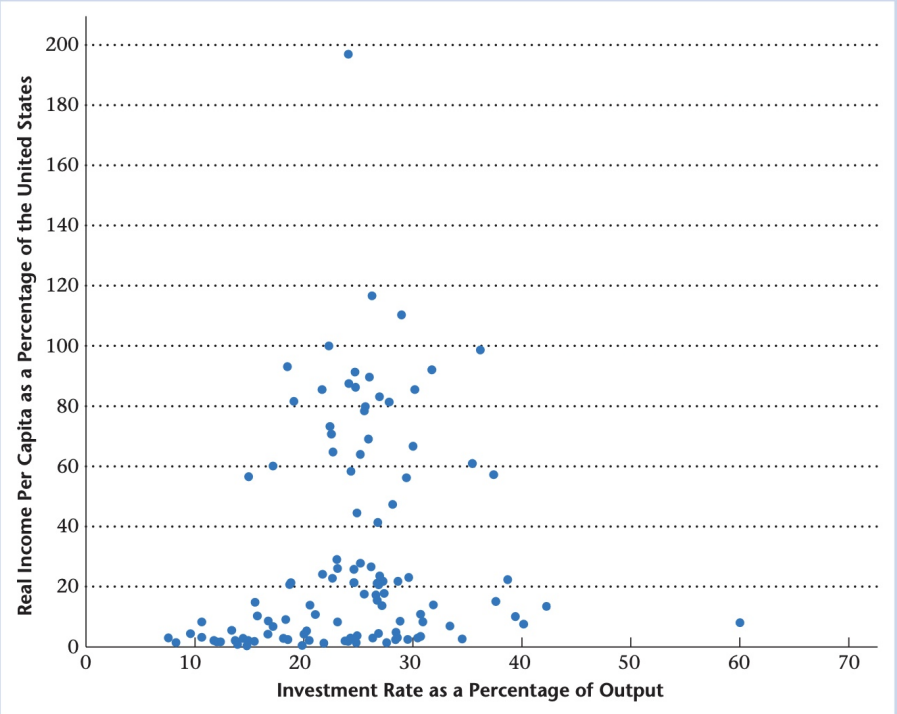
- economic/ financial barriers
- technical barriers
- regulatory/ legal / governance barriers
- human capital / skills / social barrier
- international trade & geopolitical barriers

Growth miracle could be from the removal of barrier to technology adoption.

Figure 7.2 Real Income Per Capita vs. Investment Rate

The figure shows a positive correlation across the countries of the world, between the output per capita and the investment rate.

Source: A. Heston, R. Summers, and B. Aten, *Penn World Table Version 7.0*, Center for International Comparisons of Production, Income, and Prices at the University of Pennsylvania, May 2011.



Convergence?

If income per capita in the wold is converging, we should see high growth rate in poor countries than in rich countries. i.e., negative correlation between rate of growth in income per worker and level of income per worker across countries

Figure 7.4 Growth Rate in Per Capita Income vs. Level of Per Capita Income

There is no correlation between the two variables in the figure, indicating no tendency for convergence in per capita incomes in the world over the period 1960–2007. There is much greater divergence in growth experience for the poor countries of the world than for the rich ones.

Exogenous vs. Endogenous growth models

Exogenous growth model: Solow

In Solow model, sources of growth such as z or n, per capita output or output growth, are not explained by the model (exogenous model)

Total factor productivity, z could be explained by

- R & D
- education
- training on the job

These are affected by economic environment, legal structure to support innovation, and public funding.

Endogenous Growth Model: Human Capital

Human capital are accumulated stocks of skills and knowledge. We could think of this as a **factor of efficiency of labor**

The key difference between human capital vs. physical capital

- Physical capital, such as building, machines, in Solow exhibits *diminishing returns*. Adding more capital will yield *lower marginal output*
- Human capital, such as a person's acquisition of knowledge, are *non-rivalry*. It does not prevent another person to acquire the knowledge. There is *no diminishing return to human capital*

Endogenous Growth model

Consumer

A consumer/worker can provide

- H^s: units of human capital
- 1 unit of time (no leisure)
  u of time spent working, $0 < u < 1$
  $1 - u$ of time spent accumulating human capital
- uH^s: efficiency unit of labor
- w real wage paid for each efficiency unit

Consumer

Consumption is equal to total labor earnings

$$C = wuH^s$$

A consumer cannot save, but can trade-off with future human capital.

Human capital is equivalent to future consumption

$$H^{s'} = b(1-u)H^s$$

b captures efficiency of human capital accumulation technology, $b > 0$

Firm

No physical capital in this model Output:

$$Y = zuH^d$$

where

- z is the marginal product of efficiency units of labor
- uH^d is the input of efficiency units of labor into production

Production is constant return to scale to the human capital

Firm

Firm profits

$$\begin{aligned}\pi &= y - wuH^d \\ &= zuH^d - wuH^d \\ &= (z - w)uH^d\end{aligned}$$

- if $z - w < 0, \pi < 0$: negative profit. Firms will not produce
- if $z - w > 0, \pi > 0$: positive profit. Firms will hire ∞ units of labor
- if $z = w, \pi = 0$: zero profit. Firm demand curve is perfectly elastic, always hire all available efficiency units of labor at $w = z$.

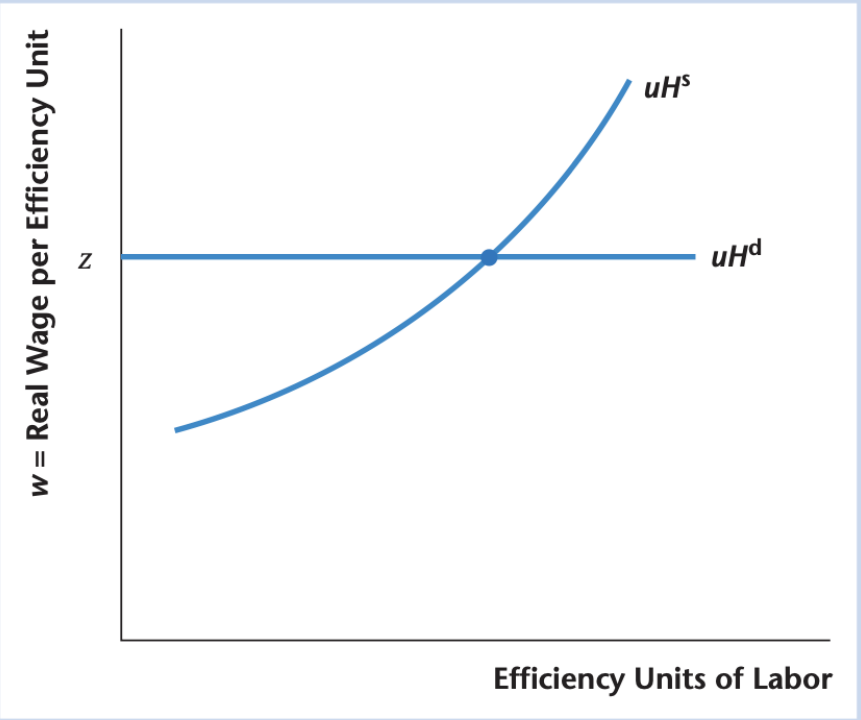
Firm

Therefore, to satisfy the market clearing condition, equilibrium real wage, w, is equal to marginal product of efficiency units of labor z

$$wH^s = zH^d$$

Figure 8.5 Determination of the Equilibrium Real Wage in the Endogenous Growth Model

The figure shows the demand and supply of efficiency units of labor in the endogenous growth model. The equilibrium wage is z, the constant marginal product of efficiency units of labor.



Competitive Equilibrium

Market always clear at

$$w = z$$

Supply = Demand:

$$uH^s = uH^d.$$

Therefore,

$$H^s = H^d = H.$$

And consumers consume all earnings, there is no saving:

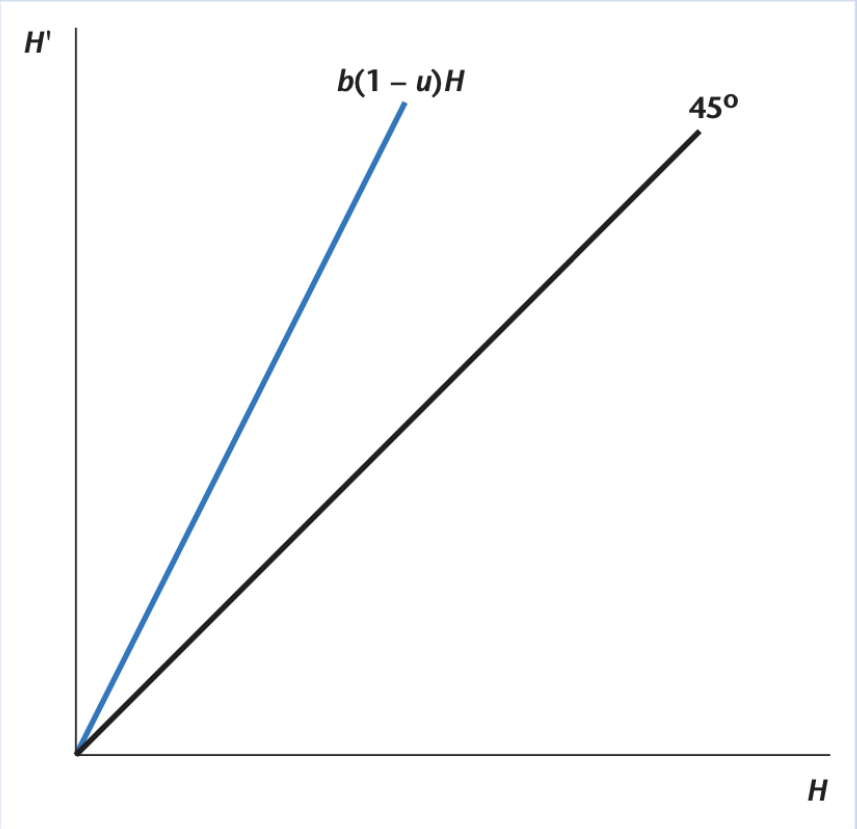
$$c = zuH.$$

Human capital dynamic

$$H' = b(1-u)H$$

Figure 8.6 Human Capital Accumulation in the Endogenous Growth Model

The colored line shows the quantity of future human capital H' as a function of current human capital H. As drawn $H' > H$ for any H, so human capital continues to increase forever.



Human capital law of motion

$$H' = b(1-u)H$$

if $b(1-u) > 1$, then $H' > H$

The growth of human capital is equal to

$$\frac{H'}{H} - 1 = b(1-u) - 1$$

- $\frac{H'}{H} - 1$ is increasing in b
- $\frac{H'}{H} - 1$ is increasing in u

Therefore, countries with better education system should experience higher rate of growth in human capital.

Consumption growth rate

Consumption growth can be written as

$$\begin{aligned}\frac{C'}{C} - 1 &= \frac{uzH'}{uzH} - 1 \\ &= \frac{H'}{H} - 1 \\ &= b(1-u) - 1\end{aligned}$$

Therefore, consumption growth rate is equal to human capital growth rate and output growth rate.

Endogenous growth

Summary of key concepts

- Endogenous growth is determined by b and u
- Key elements can led to unbounded growth, i.e., non diminishing return to scale in human capital.
- Production function is constant return to scale (CRS) to human capital, therefore output also has unbounded growth
- The non-diminishing return to human capital is motivated by the fact that *knowledge and skills* are **non-rivalry**
- Marginal product of human capital does not fall as human capital increases

Policy & Growth

Government can affect growth via policy

Question *What policy can influence human capital accumulation?*

- b : *human capital efficiency*
- $1 - u$: *time spent in human capital accumulation*

Decreases in u

If government offers tax exemption for education expenses, what happened to human capital as u decreases?

Decreases in u

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If $u \downarrow$, then $\frac{H'}{H} - 1 \uparrow$

consumption?

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Consumption growth?

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If $u \downarrow$, then $\frac{H'}{H} - 1 \uparrow$

consumption?

$$C = zuH$$

if $u \downarrow$, then $C \downarrow$

Consumption growth?

$$\frac{C'}{C} - 1 = \frac{zuH'}{zuH} - 1$$

C grows the same as H at rate $b(1-u) - 1$

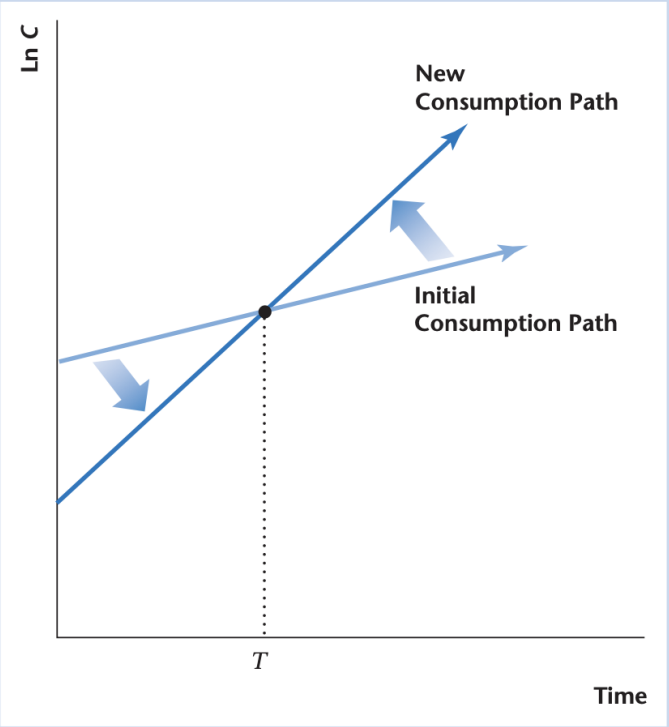
As $u \downarrow$, $C \downarrow$ and $C' \uparrow$

Decreases in u

Which consumption path consumers prefer depends on *how patient* they are. How do consumers favor *current consumption vs. future consumption*?

Figure 8.7 Effect of a Decrease in u on the Consumption Path in the Endogenous Growth Model

The figure shows the effect of a decrease in u, which increases the fraction of time spent accumulating human capital each period. The growth path for consumption (consumption is equal to income) pivots; thus, there is a short-run decrease in consumption, but consumption is higher in the long run.



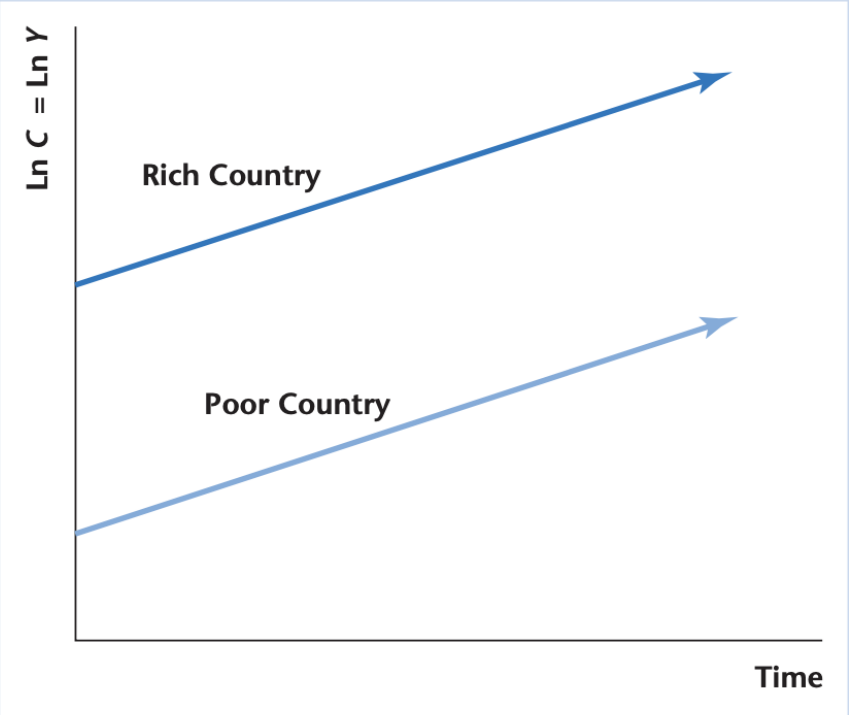
Exercise

If b, human capital accumulation efficiency changes, what happens to human capital, consumption, and consumption growth?

Convergence?

Figure 8.8 No Convergence in the Endogenous Growth Model

In the endogenous growth model, two identical countries that differ only according to their initial incomes never converge.



Conclusions and takeaways

Concluding remarks

- Human capital could explain divergence in economic growth
- However, there are things output do not account for: leisure, population health

Reading: Jones and Klenow "Beyond GDP? Welfare across countries.
Example: comparing France vs. US

Current issues labor market

- High and unmet demand for high skilled workers
- Disruption from rapid changes in technology
- Import competition from less developed country
- Wage gap between high and low skilled
- Stagnation in wages of mid-skilled workers