

## Problem set 3: Convexity, Derivatives

ECN594 2017

1. Definiteness: Sundaram Chapter 1: 62; 63 a) to d).
2. Let  $f_1, \dots, f_k$  be concave functions, each defined on  $S$ . Let  $a_1, \dots, a_k \in \mathfrak{R}_+$ . Show that  $a_1 f_1 + \dots + a_k f_k$  is a concave function on  $S$ .
3. Let  $D \subset \mathbb{R}^n$  be a convex set. Let  $f : D \rightarrow \mathbb{R}$  be differentiable function. Show that the following are equivalent:
  - $f$  is concave on  $D$ .
  - $f(y) - f(x) \leq Df(x)(y - x)$  for all  $x, y \in D$
  - $\{Df(y) - Df(x)\}(y - x) \leq 0$  for all  $x, y \in D$